# Spatio-Angular Convolutions for Super-resolution in Diffusion MRI

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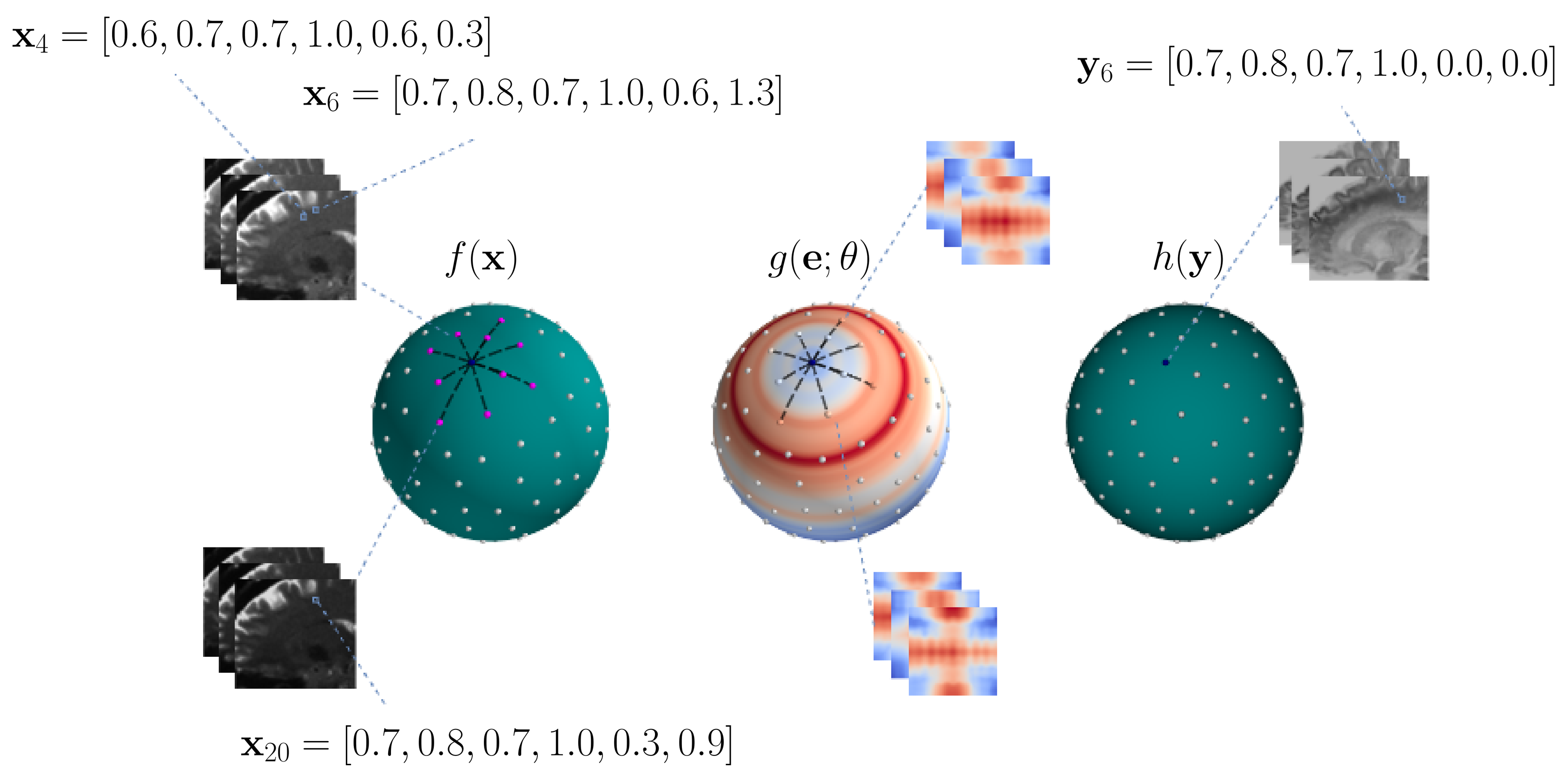
## Convolutions in Q-Space




Parametric continuous convolutions[3] extends the discrete convolution operation into the continuous domain by replacing the convolutional weights tensor with a continuous function $g$.

## Parametric Continuous Convolution Operation

**Parametric Continuous Convolution**

$$h_{k,j}(\mathbf{y}_j) = \sum_{c=1}^{C} \sum_{i=1}^{N} f_{c,i}(\mathbf{x}_i) g_{c,k}(\mathbf{e}; \boldsymbol{\theta})$$

$C$ and $K$ denote the input and output feature dimensions respectively.

**Coordinate Embedding**

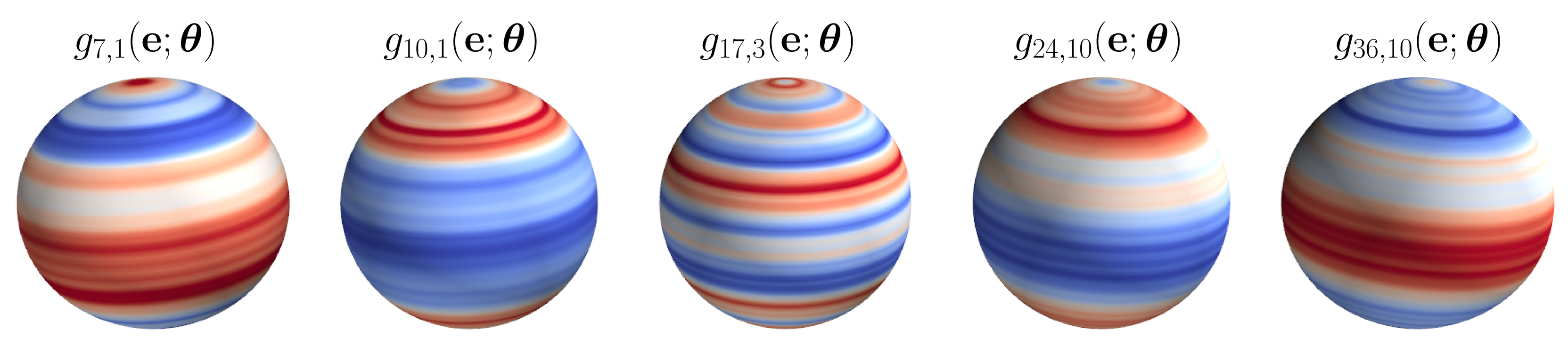
$$\mathbf{e} = [\boldsymbol{\gamma}(p_1), \boldsymbol{\gamma}(p_2), \cdots, \boldsymbol{\gamma}(p_E)]^\top$$

$$\boldsymbol{\gamma}(p_m) = [\sin(2^0 \pi p_m), \cos(2^0 \pi p_m), \cdots, \sin(2^{L-1} \pi p_m), \cos(2^{L-1} \pi p_m)]$$

$p_m$ is the $m$th component of the coordinate vector $\mathbf{p} \in \mathbb{R}^E$ given by

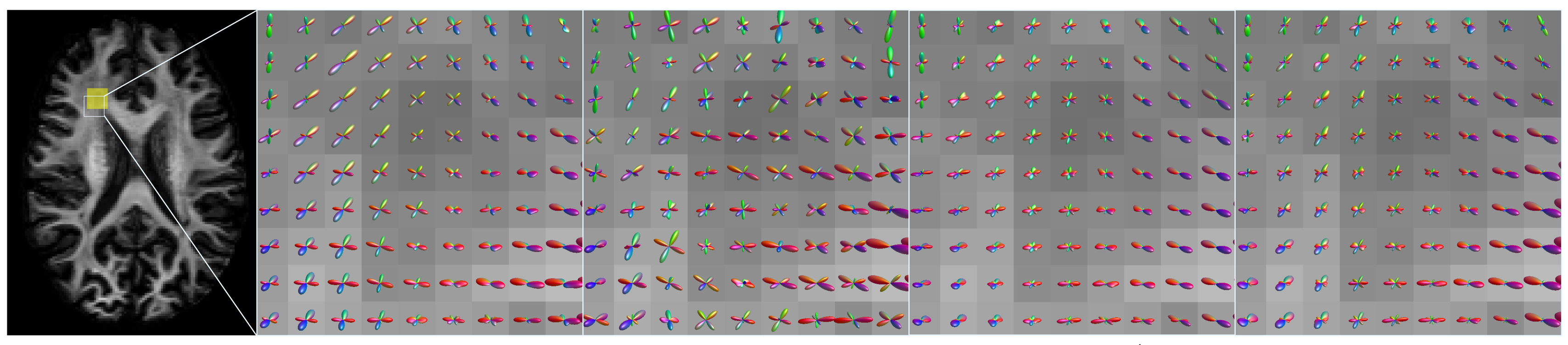
$$\mathbf{p} = [u_i - u_j, v_i - v_j, w_i - w_j, \rho_i - \rho_j, d_r]$$

## Sampled Kernel Weights




Visualisation of kernel weights sampled from $g_{c,k}(\mathbf{e}; \boldsymbol{\theta})$. Spatial coordinate components in $\mathbf{e}$ are kept constant, whilst angular components vary.

## Fibre Orientation Distribution



(a) ROI (b) High Resolution (c) Low Resolution (d) RCNN[1] (e) PCCNN-Bv-Sp

Single-shell angular super-resolution with $q_{\text{in}} = 6$ in $b = 1000\,\text{s/mm}^2$.

| | $q_{\text{in}} = 6$ | | $q_{\text{in}} = 10$ | | $q_{\text{in}} = 20$ | |
|---|---|---|---|---|---|---|
| | FOD ACC ↑ | AFD AE ↓ | FOD ACC ↑ | AFD AE ↓ | FOD ACC ↑ | AFD AE ↓ |
| Lowres | $0.653 \pm 0.008$ | $0.157 \pm 0.014$ | $0.724 \pm 0.008$ | $0.119 \pm 0.012$ | $0.757 \pm 0.010$ | $0.086 \pm 0.011$ |
| FOD-Net[5] | $\mathbf{0.743 \pm 0.008}$ | $0.087 \pm 0.005$ | $0.767 \pm 0.006$ | $\mathbf{0.072 \pm 0.004}$ | $0.776 \pm 0.008$ | $\mathbf{0.062 \pm 0.004}$ |
| RCNN[1] | $0.685 \pm 0.010$ | $\mathbf{0.087 \pm 0.006}$ | $0.749 \pm 0.009$ | $0.080 \pm 0.006$ | $0.765 \pm 0.010$ | $0.079 \pm 0.006$ |
| PCCNN | $0.658 \pm 0.009$ | $0.090 \pm 0.005$ | $0.753 \pm 0.008$ | $0.077 \pm 0.005$ | $0.792 \pm 0.009$ | $0.068 \pm 0.006$ |
| PCCNN-Bv | $0.681 \pm 0.010$ | $0.091 \pm 0.005$ | $\mathbf{0.770 \pm 0.011}$ | $0.080 \pm 0.006$ | $\mathbf{0.807 \pm 0.011}$ | $0.074 \pm 0.006$ |
| PCCNN-Bv-Sp | $0.675 \pm 0.013$ | $0.089 \pm 0.005$ | $0.766 \pm 0.014$ | $0.075 \pm 0.006$ | $0.798 \pm 0.015$ | $0.067 \pm 0.006$ |

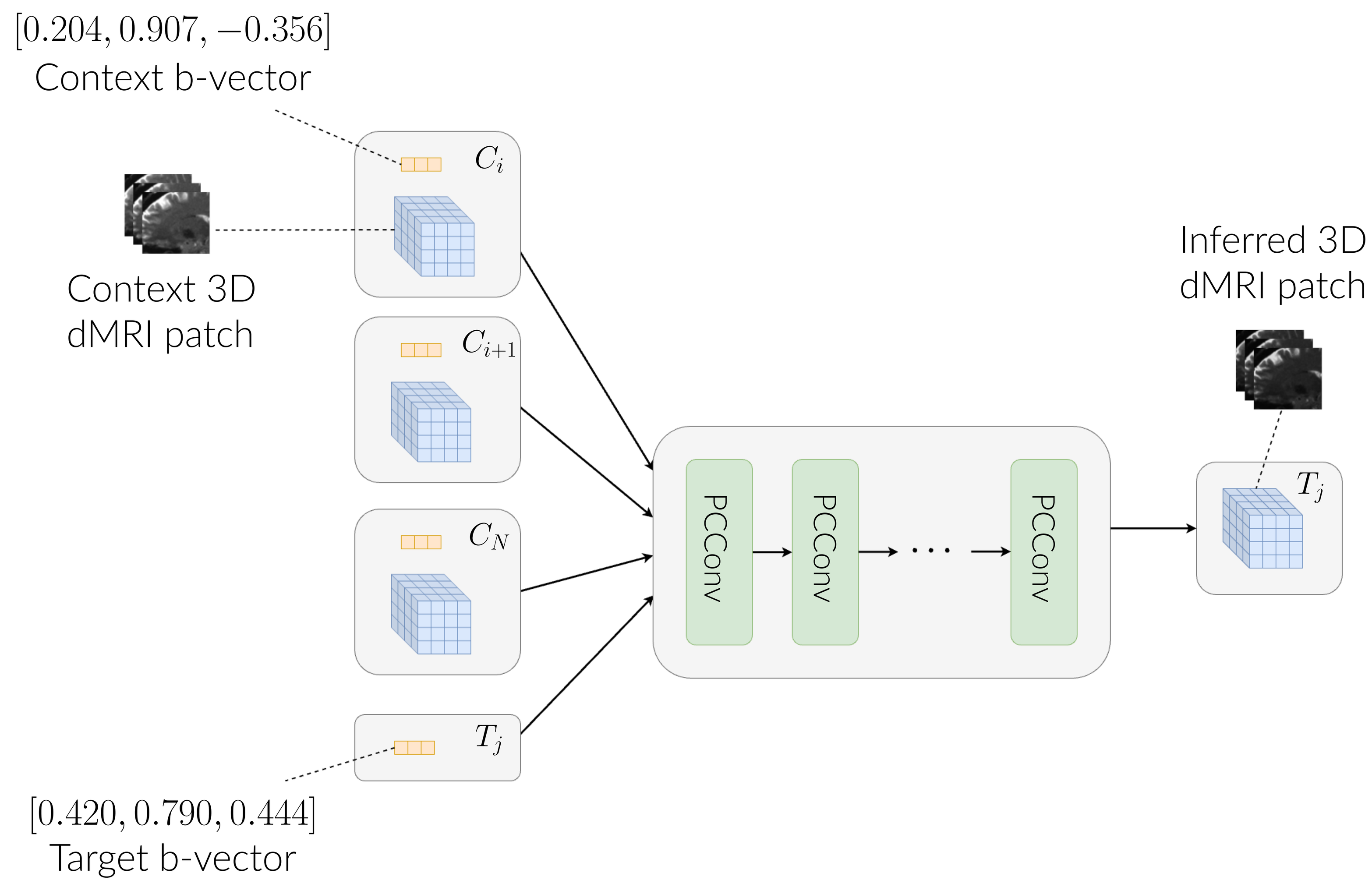
Mulit-shell angular super-resolution with varying input angular dimension size $q_{\text{in}}$ for fibre orientation distrubution (FOD) angular correlation coefficient (ACC) and apparent fibre density (AFD) absolute error (AE).

## Parameter Count

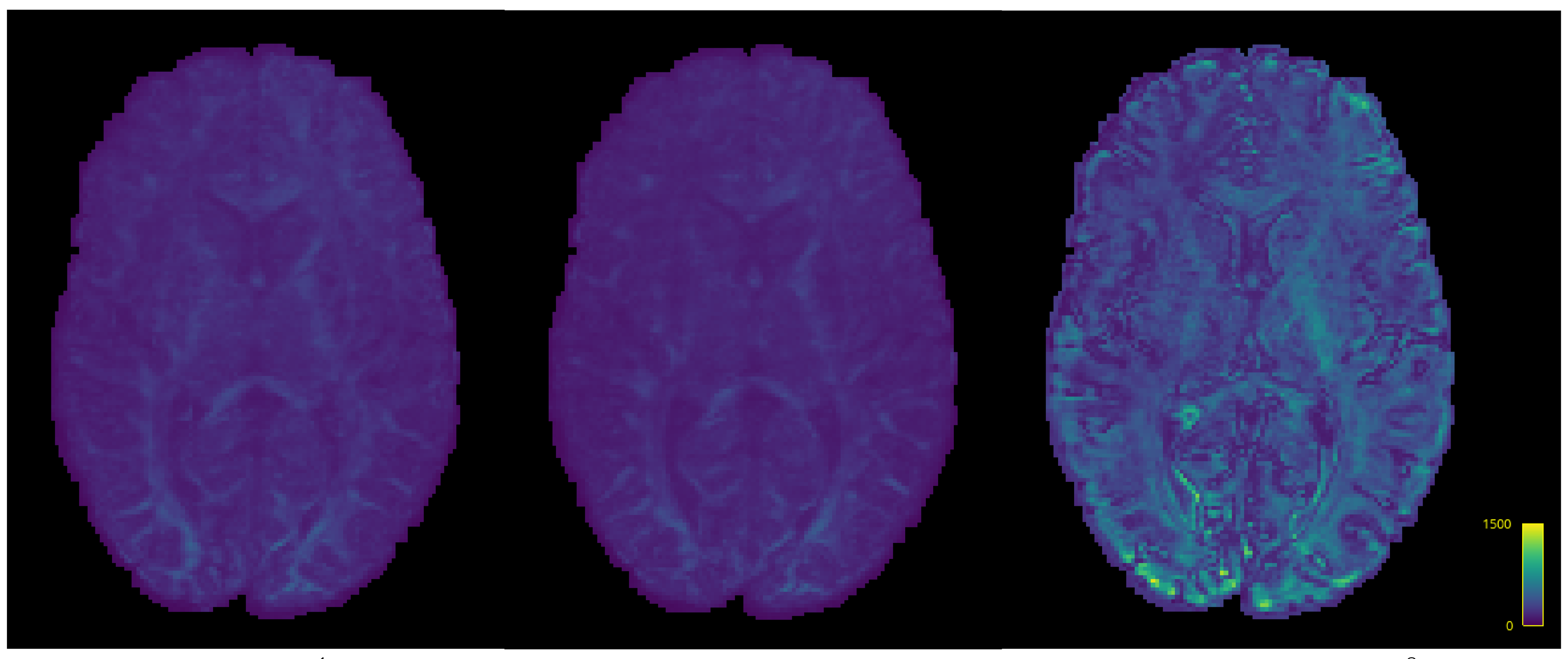
| Model | Parameters |
|---|---|
| SR-q-DL[4] | 0.52 |
| PCCNN | 0.77 |
| PCCNN-Bv | 0.79 |
| PCCNN-Bv-Sp | 0.85 |
| RCNN[1] | 6.82 |
| Q-Space CGAN[2] | 13.60 |
| FOD-Net[5] | 48.17 |

Total number of parameters, in millions, of different angular super-resolution models.

## PCCNN Angular Super-resolution Model

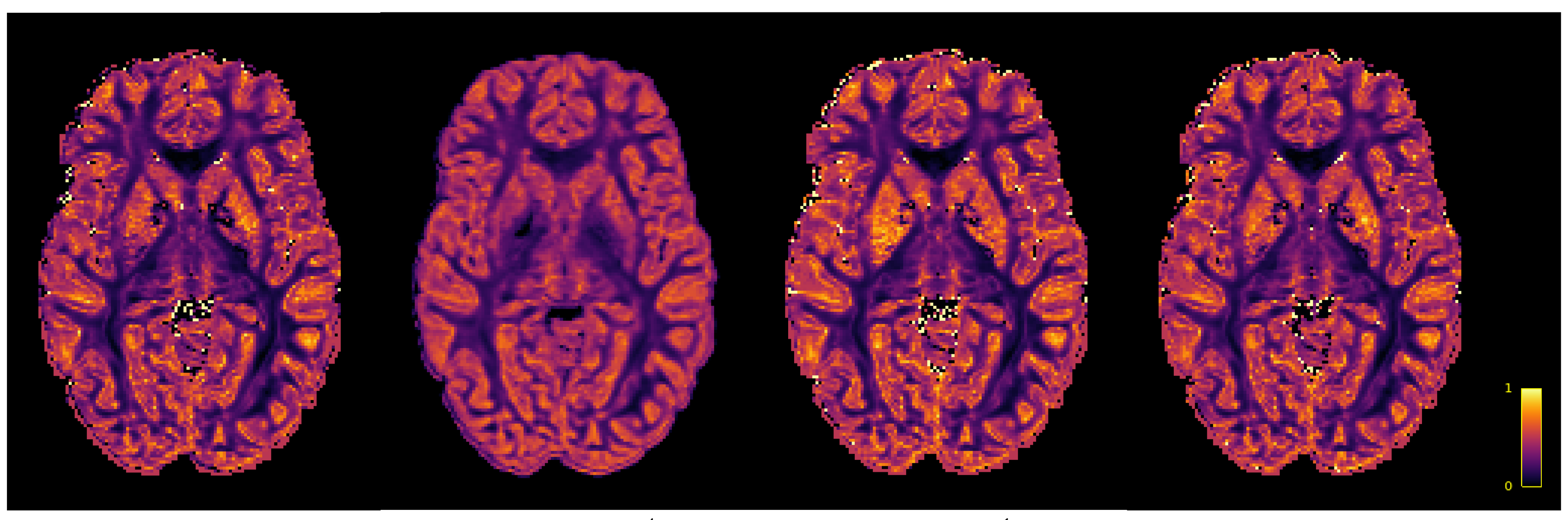



## Multi-shell dMRI Mean Absolute Error




(a) RCNN[1] (b) PCCNN-Bv-Sp (c) Q-space CGAN[2]

## Orientation Dispersion




(a) High Resolution (b) SR-q-DL[4] (c) RCNN[1] (d) PCCNN-Bv

## References

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[3] Shenlong Wang, Simon Suo, Wei-Chiu Ma, Andrei Pokrovsky, and Raquel Urtasun. Deep parametric continuous convolutional neural networks. In *Proceedings of the IEEE conference on computer vision and pattern recognition*, pages 2589–2597, 2018.

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[5] Rui Zeng, Jinglei Lv, He Wang, Luping Zhou, Michael Barnett, Fernando Calamante, and Chenyu Wang. Fod-net: A deep learning method for fiber orientation distribution angular super resolution. *Medical Image Analysis*, 79:102431, 2022.



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